A dynamic pricing model for unifying programmatic guarantee and real-time bidding in display advertising[1]

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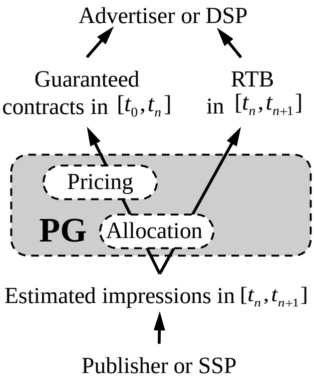
[1]The Best Paper Award in ADKDD'14, New York City, NY, USA
(in ACM proceedings database: dl.acm.org/citation.cfm?id=2648585)

PG and RTB – currently two independently processes



$[t_0, t_n]$ is the time period to sell the guaranteed impressions that will be created in future period $[t_n, t_{n+1}]$

Motivation

There is need of **a price and allocation engine** that brings automation into PG and connects RTB

- Both PG & RTB are growing rapidly:
  \$3.9bn for RTB, \$3.5bn for programmatic guaranteed (PG), US, 2014
  \$10.5bn for RTB, \$6.5bn for PG, US, 2017 projected[2]
- They both have great potential:
  \$42.78bn for online advertising, US, 2013 FY[3]

[2]MAGNA GLOBAL Ad Forecasts: Programmatic Buying Reaching a Tipping Point, 2014
[3]IAB Internet Advertising Revenue Report, 2014

Objective function

The optimization problem can be expressed as

$$\textbf{max} \quad \Bigg\{ \underbrace{\int_0^T (1-\omega\kappa)p(\tau)\theta(\tau,p(\tau))f(\tau)d\tau}_{\substack{G = \text{Expected total revenue from guaranted selling} \\ \text{minus expected penalty of failling to delivery}}} + \underbrace{\left(S - \int_0^T \theta(\tau,p(\tau))f(\tau)d\tau\right)\phi(\xi)}_{H = \text{Expected total revenue from RTB}} \Bigg\},$$

$$\textbf{s.t.} \qquad p(0) = \begin{cases} \phi(\xi) + \lambda\psi(\xi), & \text{if } \pi(\xi) \geq \phi(\xi) + \lambda\psi(\xi), \\ \pi(\xi), & \text{if } \pi(\xi) < \phi(\xi) + \lambda\psi(\xi), \end{cases}$$

where

$$\xi = \frac{\text{Remaining demand in } [t_n, t_{n+1}]}{\text{Remaining supply in } [t_n, t_{n+1}]} = \frac{Q - \int_0^T \theta(\tau,p(\tau))f(\tau)d\tau}{S - \int_0^T \theta(\tau,p(\tau))f(\tau)d\tau}.$$

Distribution of bids in RTB

1. **Log-normal distribution**: $X \sim \mathbf{LN}(\mu, \sigma^2)$
   The expected per impression payment price from a second-price auction is

$$\phi(\xi) = \int_0^\infty x\xi(\xi-1)g(x)\Big(1-F(x)\Big)\Big(F(x)\Big)^{\xi-2}dx,$$

   where

$$g(x) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{(\ln(x)-\mu)^2}{2\sigma^2}}, \quad F(x) = \frac{1}{2} + \frac{1}{\sqrt{\pi}} \int_0^{\frac{\ln(x)-\mu}{\sqrt{2}\sigma}} e^{-z^2} dz.$$

2. **Empirical method**
   Robust Locally Weighted Regression (see Algorithm 1)

Purchase behaviour

1. **One buys less if an inventory is expensive**
   Given τ and $0 \leq p_1 \leq p_2$, then $\theta(\tau, p_1) \geq \theta(\tau, p_2)$, s.t. $\theta(\tau, 0) = 1$.
2. **One buys less if it is early**
   Given p and $0 \leq \tau_2 \leq \tau_1$, then $\theta(\tau_2, p) \geq \theta(\tau_1, p)$.

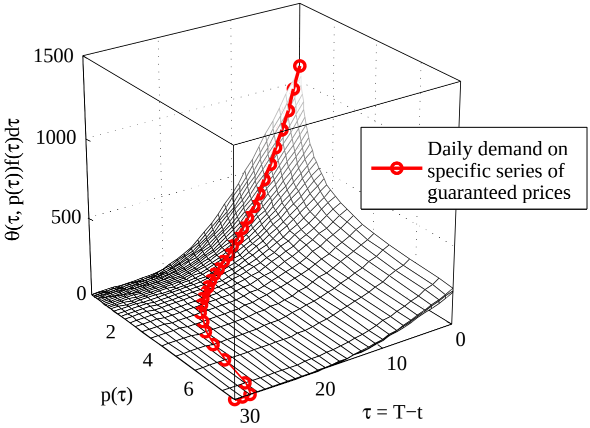
We adopt the functional forms of demand:

$$\theta(\tau, p(\tau)) = e^{-\alpha p(\tau)(1+\beta\tau)},$$
$$f(\tau) = \zeta e^{-\eta\tau},$$

where α is the level of price effect, β and η are the levels of time effect, and the demand density rises to a peak ζ on the delivery date, so that $\theta(\tau, p(\tau))f(\tau)d\tau$ is the number of advertisers who will buy guaranteed impressions at $p(\tau)$.

Demand surface $\theta(\tau, p(\tau))f(\tau)d\tau$

$\alpha = 1.85,$
$\beta = 0.01,$
$\zeta = 2000,$
$\eta = 0.01,$
$T = 30.$



Solution

The objective function is solved by Algorithm 2, in which the optimal guaranteed price can be described as follows:

$$p(\tau) = \frac{\widetilde{\lambda}}{1 - \omega\kappa} + \frac{1}{\alpha(1 + \beta\tau)}.$$

The notation $\widetilde{\lambda}(\alpha, \beta, \zeta, \eta, \omega, \kappa, \gamma_i S)$ represents the dependency relationship among $\widetilde{\lambda}$ and other parameters.

Solution

Algorithm 2:

function PGSolve$(\alpha, \beta, \zeta, \eta, \omega, \kappa, \lambda, S, Q, T)$

$\boldsymbol{t} \leftarrow [t_0, \cdots, t_n]$, $0 = t_0 < t_1 < \cdots < t_n = T$.

$\boldsymbol{\tau} \leftarrow T - \boldsymbol{t}$, $m \leftarrow \#$ of simulations.

loop $i \leftarrow 1$ to m

$\gamma_i \leftarrow$ **RandomUniformGenerate**$([0,1])$

$\int_0^T \theta(\tau, p(\tau)) f(\tau) d\tau \leftarrow \gamma_i S$

$\xi_i \leftarrow (Q - \gamma_i S)/(S - \gamma S)$

$H_i \leftarrow (1 - \gamma_i) S \phi(\xi_i)$

$G_i \leftarrow \int_0^T (1 - \omega\kappa) p(\tau) \theta(\tau, p(\tau)) f(\tau) d\tau$

$$\boldsymbol{p}_i \leftarrow \mathop{\mathbf{arg\,max}}\ G_i,$$

$$\textbf{s.t.} \int_0^T \theta(\tau, p(\tau)) f(\tau) d\tau = \gamma_i S,$$

$$p(0) = \begin{cases} \phi(\xi_i) + \lambda\psi(\xi_i), \text{if } \pi(\xi_i) \geq \phi(\xi_i) + \lambda\psi(\xi_i), \\ \pi(\xi_i), \text{if } \pi(\xi_i) < \phi(\xi_i) + \lambda\psi(\xi_i). \end{cases}$$

$R_i \leftarrow \mathbf{max}\, G_i + H_i$

end loop

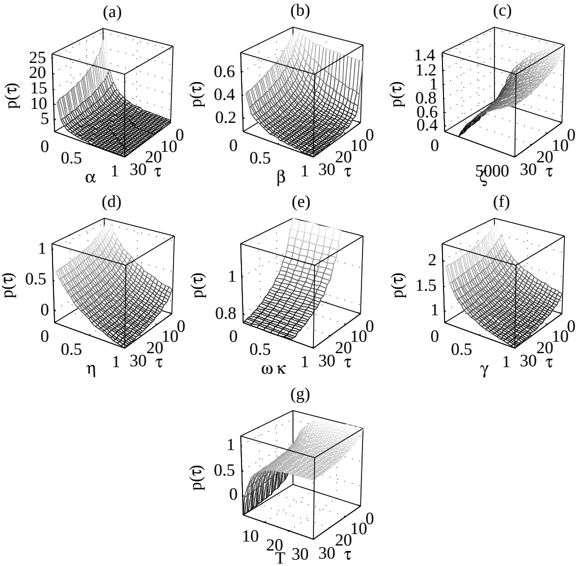
$\gamma^* \leftarrow \mathbf{arg\,max}_{\gamma_i \in \Omega(\gamma)} \{R_1, \ldots, R_m\}$

$\boldsymbol{p}^* \leftarrow \mathbf{arg\,max}_{\boldsymbol{p}_i \in \Omega(\boldsymbol{p})} \{R_1, \ldots, R_m\}$

return $\gamma^*, \mathbf{p}^*$

end function

Effects of parameters



Datasets

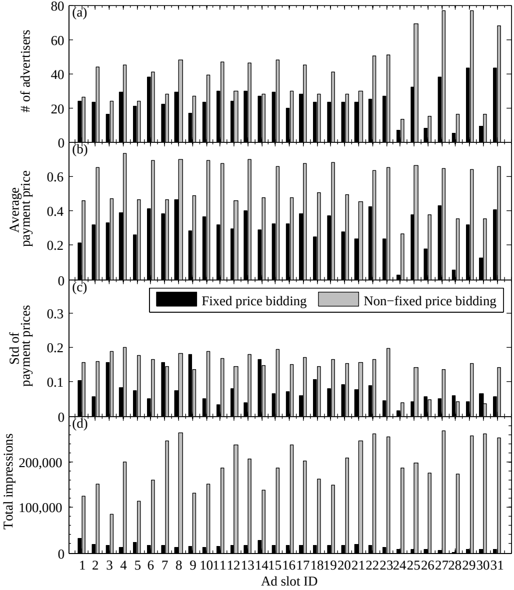
Table: Summary of RTB datasets.

Dataset	SSP	DSP
From	08/01/2013	19/10/2013
To	14/02/2013	27/10/2013
# of ad slots	31	53571
# of user tags	NA	69
# of advertisers	374	4
# of impressions	6646643	3158171
# of bids	33043127	11457419
Bid quote	USD/CPM	CNY/CPM

Table: Experimental design of the SSP dataset.

	From	To
Training set	08/01/2013	13/02/2013
Development set	08/01/2013	14/02/2013
Test set	14/02/2013	

Bidding behaviours



Bidding behaviours

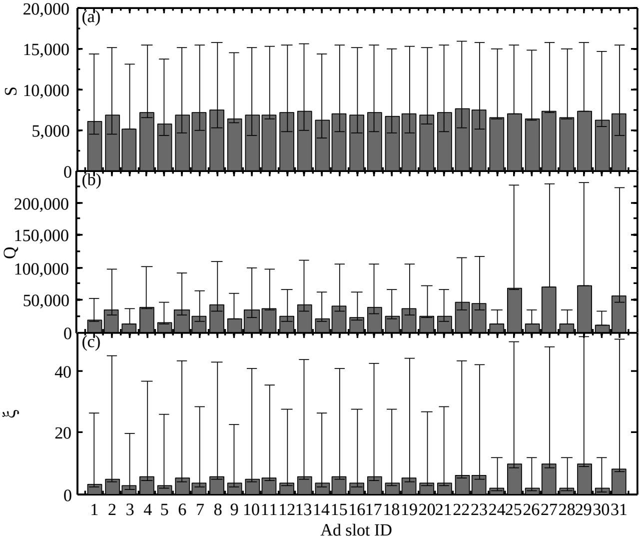
Table: Summary of the winning advertisers' statistics from the SSP dataset in the training period: the numbers in the brackets represent how many advertisers who use the combined bidding strategies.

Bidding strategy	# of advertisers	# of change imps won	Average change rate of payment prices	Ratio of payment price to winning bid
Fixed price	188 (51)	454681	188.85%	43.93%
Non-fixed price	200 (51)	6068908	517.54%	58.94%

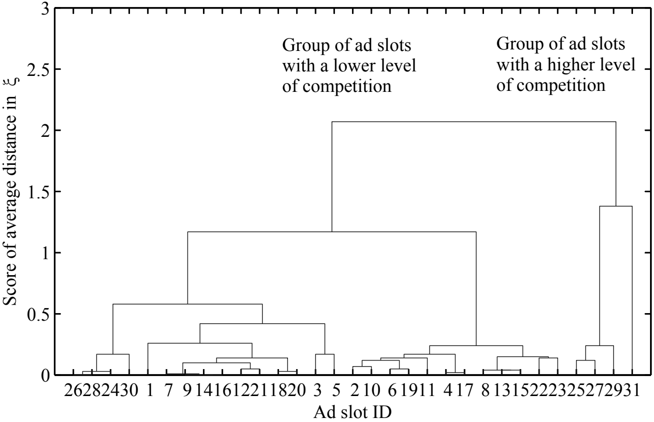
Table: Summary of advertisers' winning campaigns from the DSP dataset. All the advertisers use the fixed price bidding strategy. Each user tag contains many ad slots and an ad slot is sampled from the dataset only if the advertiser wins more than 1000 impressions from it.

Advertiser ID	# of user tags	# of ad slots	# of imps won	Average change rate of payment prices	Ratio of payment price to winning bid
1	69	635	196831	58.57%	36.07%
2	69	428	144272	58.94%	34.68%
3	69	1267	123361	79.24%	30.89%
4	65	15	3139	104.19%	22.32%

Supply and demand



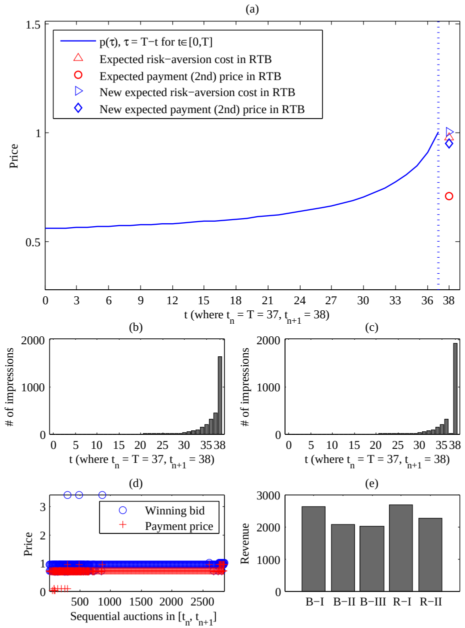
Demand per impression reflects the market competition



Empirical example 1: (AdSlot14) demand per impression 3.39

In a less competitive market:

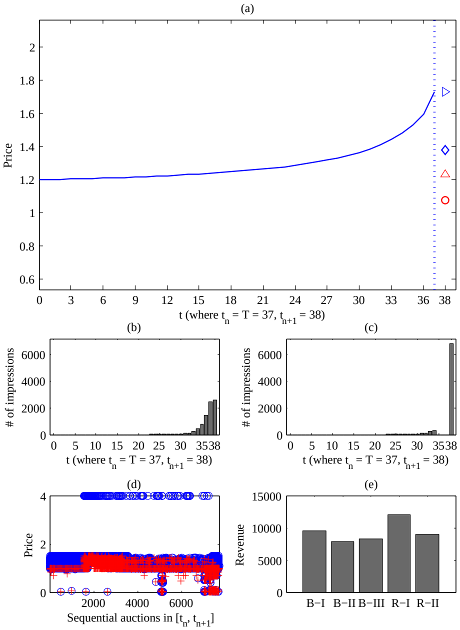
- Fewer buyers are willing to buy in advance
- Less impressions to PG (here 42.40%)
- PG prices are not expensive
- Revenue mainly contributed by RTB



Empirical example 2: (AdSlot27) demand per impression 9.63

In a competitive market:

- More buyers are willing to buy in advance
- More impressions to PG (here 66%)
- PG prices are higher
- Revenue mainly contributed by PG

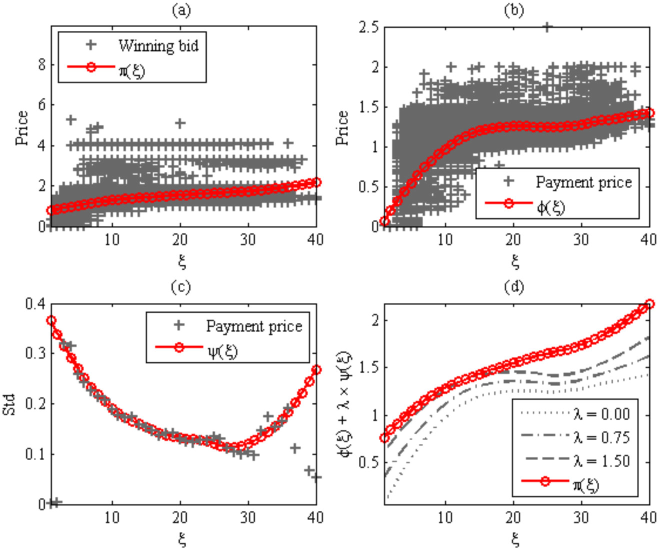


Revenue analysis

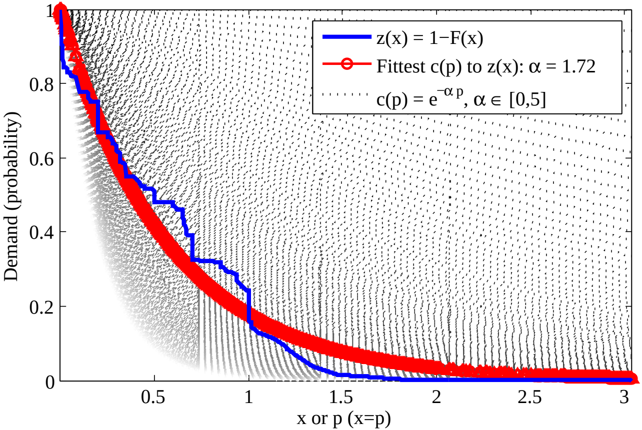
Table: Summary of revenue evaluation of all 31 ad slots in the SSP dataset.

Group of ad slots	Revenue maximization			Price discrimination	
	Estimated revenue increase	Actual revenue increase	Difference of RTB revenue between estimation & actual	Ratio of actual 2nd price reve to actual 1st price reve	Ratio of actual optimal reve to actual 1st price reve
Low competition	31.06%	8.69%	13.87%	67.05%	81.78%
High competition	31.73%	21.51%	6.23%	78.04%	94.70%

Parameter estimation 1



Parameter estimation 2



α is calculated based on the smallest RMSE between the inverse function of empirical CDF of bids $z(x) = 1 - F(x)$ and the function $c(p) = e^{-\alpha p}$

Concluding remarks

This paper proposes a mathematical model that allocates and prices the future impressions between real-time auctions and guaranteed contracts. Under conventional economic assumptions, our model shows that the two ways can be seamless combined programmatically and the publisher's revenue can be maximized via price discrimination and optimal allocation.

Thank you!

Welcome questions

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