Combining guaranteed and spot markets in display advertising: selling guaranteed page views with stochastic demand[1]

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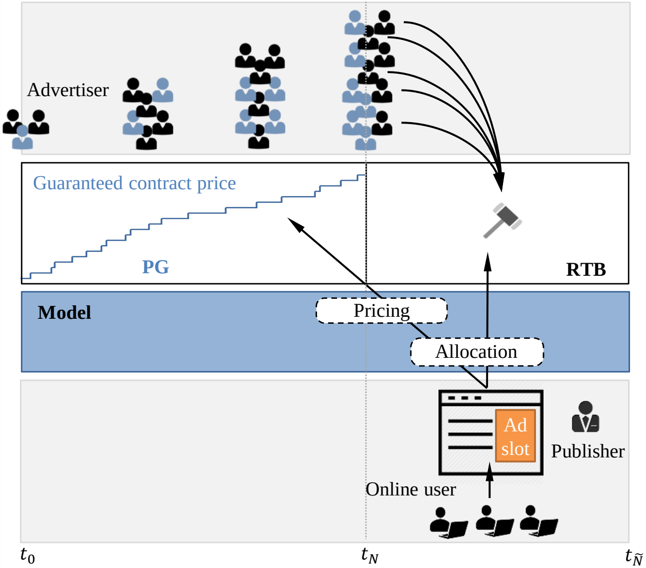
Schematic view

- Ad impressions will be created and be real-time auctioned off (i.e., RTB) in $[t_N, t_{\widetilde{N}}]$;
- They can be sold in advance via guaranteed contracts in $[t_0, t_N]$;
- Advertisers' demand of display advertising in $[t_N, t_{\widetilde{N}}]$ arrives sequentially over time in $[t_0, t_N]$;
- The unfulfilled demand will join RTB in $[t_N, t_{\widetilde{N}}]$.



Stochastic demand arrivals and purchase behaviour

Let $f(t_n)$ be the expected number of advertisers arriving in $\Delta t = t_n - t_{n-1}$, which follows a homogeneous Poisson process, then the demand for buying a guaranteed contract at time t_n can be computed as

$$\eta(t_n) = \mathbb{I}_{\{n>0\}} \sum_{i=0}^{n-1} f(t_i) \prod_{j=i}^{n-1} \Big[1 - \theta(t_j, p(t_j))\Big] + f(t_n), \tag{1}$$

where $\mathbb{I}_{\{\cdot\}}$ is an indicator function, $\sum_{i=0}^{n-1} f(t_i) \prod_{j=i}^{n-1} \big(1 - \theta(t_j, p(t_j))\big)$ computes the unfulfilled demand backlogged from the previous time periods and $\theta(t_n, p(t_n))$ is the proportion of those who want to buy an impression in advance at time t_n and at price $p(t_n)$, defined as

$$\theta(t_n, p(t_n)) = \exp\bigg\{ - \alpha p(t_n)\Big(1 + \beta(t_N - t_n)\Big)\bigg\}, \tag{2}$$

where α represents the price effect and β represents the time effect.

RTB-based terminal value

The expected revenue from RTB can be obtained as

$$\phi(\xi) = \int_{\Omega} x\xi(\xi-1)g(x)\big[1-F(x)\big]\big[F(x)\big]^{\xi-2}dx, \tag{3}$$

where x is an advertiser's bid, Ω is the range of bid, $g(\cdot)$ and $F(\cdot)$ are the density and cumulative distribution functions, respectively. Thus, $\xi(\xi-1)g(x)\big[1-F(x)\big]\big[F(x)\big]^{\xi-2}$ represents the probability that if an advertiser who bids at x is the second highest bidder, then one of $\xi-1$ other advertisers must bid at least as much as he does and all of $\xi-2$ other advertisers have to bid no more than he does.

Censored upper bound for pricing

The censored upper bound of the guaranteed contract price at time t_n can be characterised as

$$\Phi(t_n) = \min\Big\{ \underbrace{\phi(\xi(t_n)) + \delta(t_n)\psi(\xi(t_n))}_{:=\chi(t_n,\xi(t_n))}, \pi \Big\}, \tag{4}$$

where π is the expected maximum value of an impression, $\psi(\xi(t_n))$ is the standard deviation of payment prices in RTB, $\delta(t_n)$ is advertiser's risk preference and $\delta(t_n) = \zeta e^{-v t_n}$.

Revenue maximisation

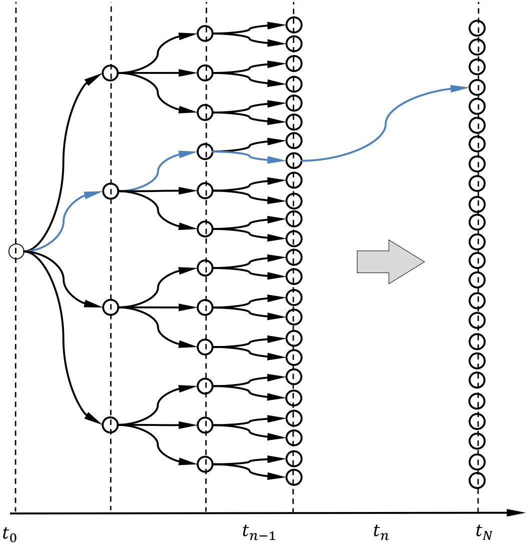
$$\max \quad R = \left\{ \underbrace{\sum_{n=0}^{N} (1 - \omega\varpi) p(t_n) \theta\big(t_n, p(t_n)\big) \eta(t_n)}_{:= R^{PG}} + \underbrace{\left[S - \sum_{n=0}^{N} \theta\big(t_n, p(t_n)\big) \eta(t_n) \right] \phi\big(\xi(t_N)\big)}_{:= R^{RTB}} \right\}, \tag{5}$$

$$\text{s.t.} \quad 0 \leq p(t_n) \leq \Phi(t_n), \text{for } n = 0, \cdots, N, \tag{6}$$

$$0 \leq \sum_{n=0}^{N} \theta\big(t_n, p(t_n)\big) \eta(t_n) \leq S. \tag{7}$$

Solution based on Knapsack problem

- Algorithms 1-2



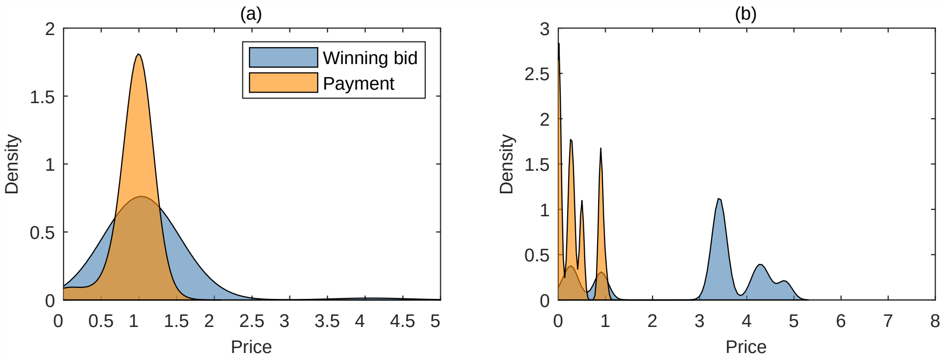
Data

A dataset from a UK SSP that contains 1,378,971 RTB campaigns for 31 different ad slots over the period from 08 January 2013 to 14 February 2013.

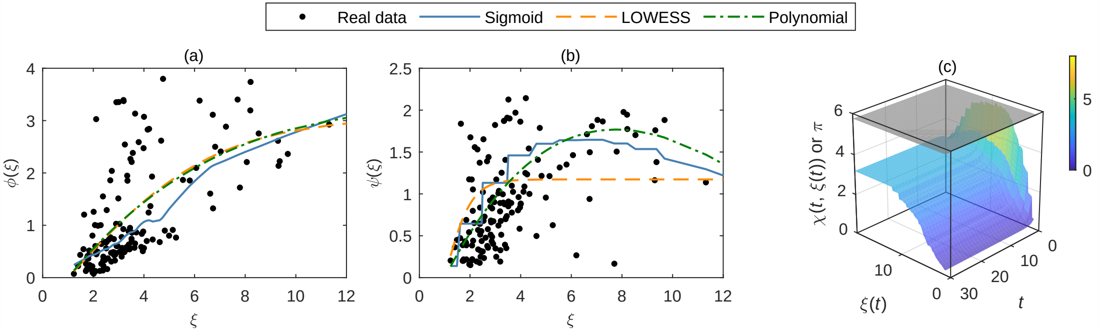
Group	1		2	
Number of ad slots	6		20	
Set	Training	Test	Training	Test
Payment price	0.98 (0.09)	0.99 (0.08)	0.73 (0.46)	0.56 (0.36)
Winning bid	1.13 (0.17)	1.1 (0.1)	2.32 (1.17)	1.84 (1.04)
ξ	8.92 (3.24)	8.15 (1.18)	3.39 (0.59)	3.51 (0.81)
Ratio of payment price to winning bid	88.95% (4.54%)	92.88% (2.15%)	32.2% (9.9%)	37.18% (10.58%)

Note: numbers in round brackets are standard deviations

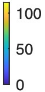
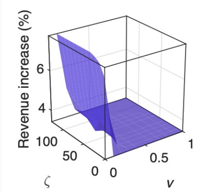
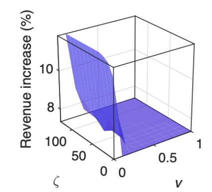
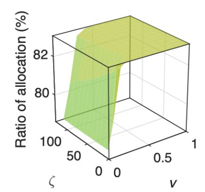
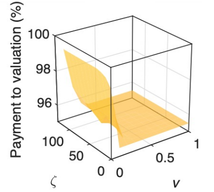
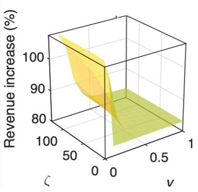
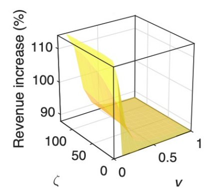
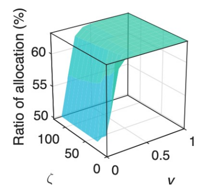
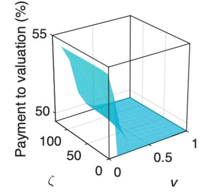
Winning bids vs payment prices



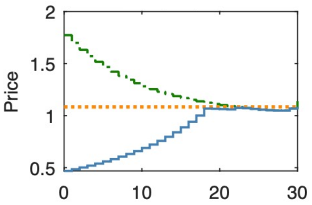
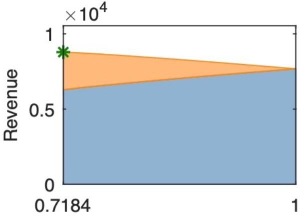
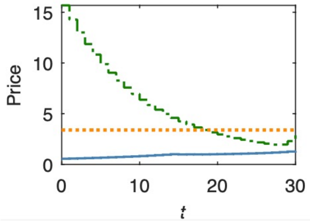
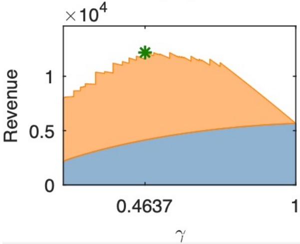
Estimating model parameters



Overall performance

	Average revenue increase of the model to the expected RTB	Average revenue increase of the model to the actual RTB	Average ratio of selling impressions in advance made by the model	Average ratio of payment to value made by the model
Competitive market (overall)				
Less competitive market (overall)				

Optimal pricing and allocation

	Pricing	Allocation
	-·-· $\chi(t, \xi(t))$ ······ π —— $p(t)$	▨ R^{PG} ▨ R^{RTB} ✱ Optimal point
Competitive market (example of ad slot 24)		
Less competitive market (example of ad slot 15)		

Thank you!

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