Multi-keyword multi-click advertisement option contracts for sponsored search[1]

Bowei Chen[†,2], Jun Wang[†], Ingemar J. Cox[†,‡], Mohan S. Kanhanhalli[♯]

[†]University College London
[‡]University of Copenhagen
[♯]National University of Singapore

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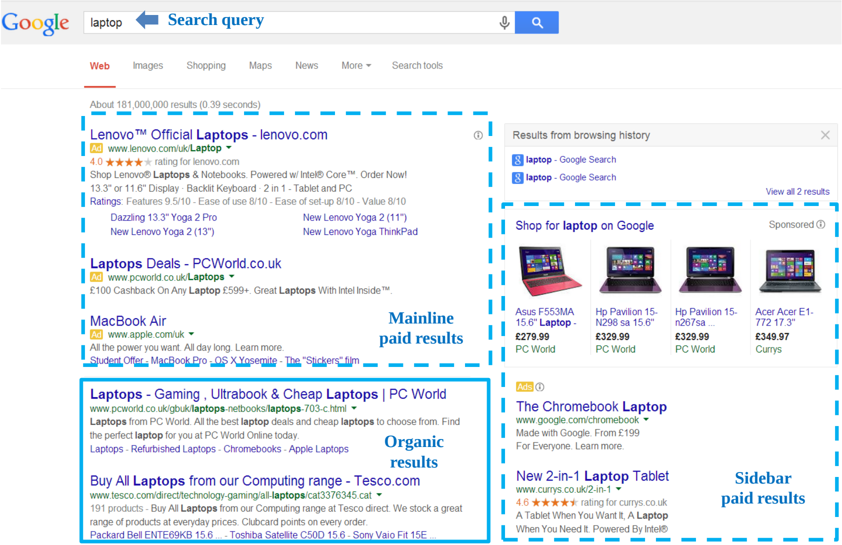
[2] Corresponding author: bowei.chen@cs.ucl.ac.uk

Sponsored search



Search query

About 181,000,000 results (0.39 seconds)

Lenovo™ Official Laptops - lenovo.com
Ad www.lenovo.com/uk/**Laptop**
4.0 ★★★★★ rating for lenovo.com
Shop Lenovo® **Laptops** & Notebooks. Powered w/ Intel® Core™. Order Now!
13.3" or 11.6" Display · Backlit Keyboard · 2 in 1 - Tablet and PC
Ratings: Features 9.5/10 - Ease of use 8/10 - Ease of set-up 8/10 - Value 8/10
Dazzling 13.3" Yoga 2 Pro — New Lenovo Yoga 2 (11")
New Lenovo Yoga 2 (13") — New Lenovo Yoga ThinkPad

Laptops Deals - PCWorld.co.uk
Ad www.pcworld.co.uk/**Laptops**
£100 Cashback On Any **Laptop** £599+. Great **Laptops** With Intel Inside™.

MacBook Air
Ad www.apple.com/uk
All the power you want. All day long. Learn more.
Student Offer - MacBook Pro - OS X Yosemite - The "Stickers" film

Mainline paid results

Laptops - Gaming , Ultrabook & Cheap Laptops | PC World
www.pcworld.co.uk/gbuk/**laptops**-netbooks/**laptops**-703-c.html
Laptops from PC World. All the best **laptop** deals and cheap **laptops** to choose from. Find the perfect **laptop** for you at PC World Online today.
Laptops - Refurbished Laptops - Chromebooks - Apple Laptops

Buy All Laptops from our Computing range - Tesco.com
www.tesco.com/direct/technology-gaming/all-**laptops**/cat3376345.cat
191 products - Buy All **Laptops** from our Computing range at Tesco direct. We stock a great range of products at everyday prices. Clubcard points on every order.
Packard Bell ENTE69KB 15.6 ... - Toshiba Satellite C50D 15.6 - Sony Vaio Fit 15E ...

Organic results

Results from browsing history
laptop - Google Search
laptop - Google Search
View all 2 results

Shop for **laptop** on Google — Sponsored

Asus F553MA 15.6" **Laptop** -	Hp Pavilion 15-N298 sa 15.6"	Hp Pavilion 15-n267sa ...	Acer Acer E1-772 17.3"
£279.99	**£329.99**	**£329.99**	**£349.97**
PC World	PC World	PC World	Currys

Ads

The Chromebook Laptop
www.google.com/chromebook
Made with Google. From £199
For Everyone. Learn more.

New 2-in-1 Laptop Tablet
www.currys.co.uk/2-in-1
4.6 ★★★★★ rating for currys.co.uk
A Tablet When You Want It, A **Laptop** When You Need It. Powered By Intel®

Sidebar paid results

Limitations of auction mechanisms in sponsored search

- Uncertainty in payment prices for advertisers;
- Volatility in the search engine's revenue;
- Weak loyalty between advertiser and search engine.

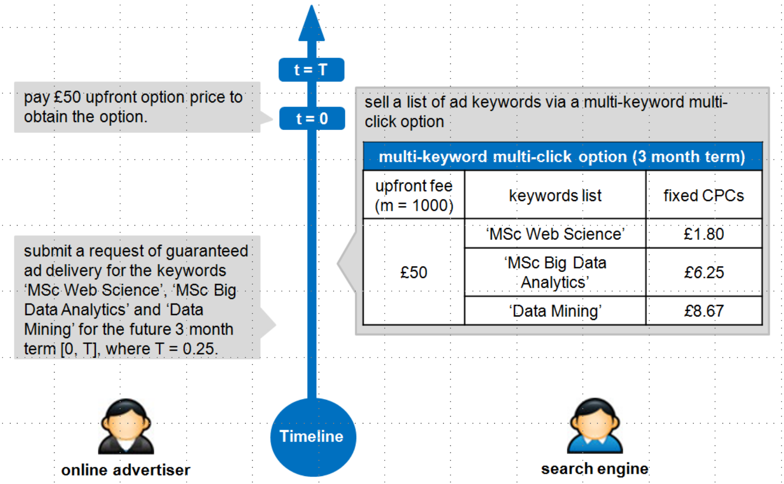
Tailored exotic options to sponsored search

A **multi-keyword multi-click ad option contract** allows its buyer to:

- Target a set of candidate keywords for a certain number of total clicks;
- Multiply exercise the option to purchase guaranteed clicks at any time on or prior to the contract expiration date;
- Switch among candidate keywords when exercise the option without paying any cost.

It has the properties of multi-asset option and multi-exercise option.

Buy and sell an ad option



pay £50 upfront option price to obtain the option.

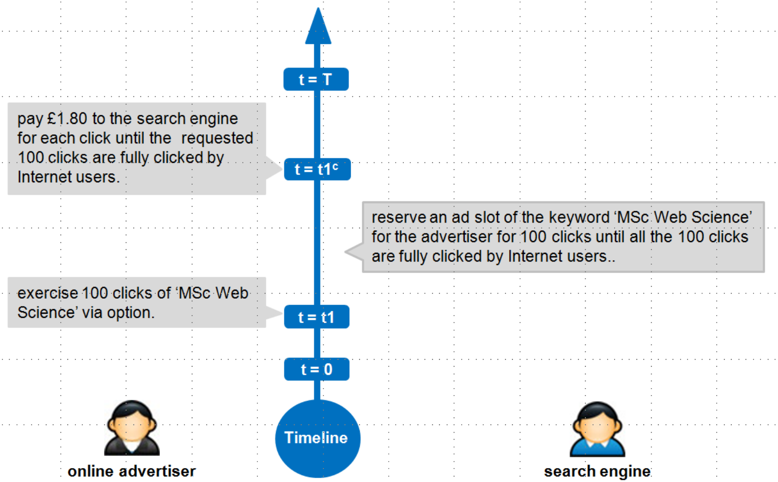
submit a request of guaranteed ad delivery for the keywords 'MSc Web Science', 'MSc Big Data Analytics' and 'Data Mining' for the future 3 month term [0, T], where T = 0.25.

sell a list of ad keywords via a multi-keyword multi-click option

multi-keyword multi-click option (3 month term)

upfront fee (m = 1000)	keywords list	fixed CPCs
£50	'MSc Web Science'	£1.80
	'MSc Big Data Analytics'	£6.25
	'Data Mining'	£8.67

Exercise an ad option



pay £1.80 to the search engine for each click until the requested 100 clicks are fully clicked by Internet users.

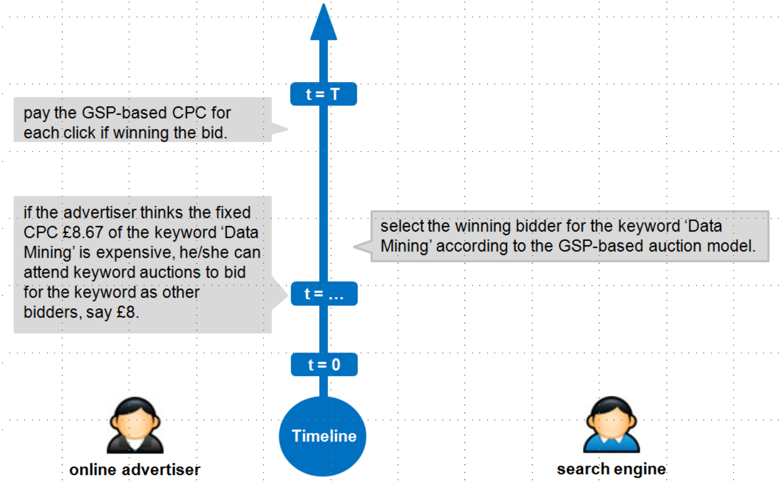
reserve an ad slot of the keyword 'MSc Web Science' for the advertiser for 100 clicks until all the 100 clicks are fully clicked by Internet users..

exercise 100 clicks of 'MSc Web Science' via option.

online advertiser

search engine

Not exercise an ad option



pay the GSP-based CPC for each click if winning the bid.

if the advertiser thinks the fixed CPC £8.67 of the keyword ‘Data Mining’ is expensive, he/she can attend keyword auctions to bid for the keyword as other bidders, say £8.

select the winning bidder for the keyword ‘Data Mining’ according to the GSP-based auction model.

Benefits

Advertiser	Search engine
Secure ad service delivery	Obtain upfront income in advance
Reduce uncertainty from auctions	Have a stable & increased revenue
Cap advertising costs	Increase advertisers' loyalty online

Option pricing building blocks

- Underlying stochastic model
- Option payoff formulation
- Option pricing framework and solution

Multivariate geometric Brownian motion

The keyword K_i's spot market CPC can be described as follows

$$dC_i(t) = \mu_i C_i(t)dt + \sigma_i C_i(t)dW_i(t), \quad i = 1, \ldots, n,$$

where μ_i and σ_i are the drift and volatility, respectively, and $W_i(t)$ is a standard Brownian motion satisfying the conditions:

$$\begin{aligned} \mathbb{E}(dW_i(t)) &= 0, \\ \text{var}(dW_i(t)) &= \mathbb{E}(dW_i(t)dW_i(t)) = dt, \\ \text{cov}(dW_i(t), dW_j(t)) &= \mathbb{E}(dW_i(t)dW_j(t)) = \rho_{ij}dt, \end{aligned}$$

where ρ_{ij} is the correlation coefficient between the ith and jth keywords, such that $\rho_{ii} = 1$ and $\rho_{ij} = \rho_{ji}$. The correlation matrix is denoted by $\mathbf{\Sigma}$, so that the covariance matrix is $\boldsymbol{M\Sigma M}$, where $\boldsymbol{M}$ is the matrix with σ_i along the diagonal and zeros everywhere else.

Option value and payoff

The value of an m-click ad option at time t is equal to m number of 1-click ad option:

$$V(t, \boldsymbol{C}(t); T, \boldsymbol{F}, m) = mV(t, \boldsymbol{C}(t); T, \boldsymbol{F}, 1).$$

If exercised, we have

$$V(t, \boldsymbol{C}(t); T, \boldsymbol{F}, 1) = \Phi(\mathbf{C}(t)) := \max\{C_1(t) - F_1, \ldots, C_n(t) - F_n, 0\},$$

where $\mathbf{F}$ is a vector of exercise prices for candidate keywords, T is the expiration date, and $\Phi(\mathbf{C}(t))$ is the option payoff at time t.

No early exercise property

Since $e^{-rt}\Phi\big(\boldsymbol{X}(t)\big)$ is a sub-martingale under the risk-neutral probability measure $\mathbb{Q}$ (see Appendix A), the proposed ad option can be priced as same as its European structure, focusing on the payoff on the contract expiration date.

Option pricing framework

The option price π_0 (i.e., the option value at time 0) can be obtained as follows (see Appendix B for the derivation):

$$\begin{aligned}\pi_0 &= \mathbb{E}[\Phi(\mathbf{C}(T)) \mid \mathbb{F}_0] \\ &= m e^{-rT} (2\pi T)^{-\frac{n}{2}} |\mathbf{\Sigma}|^{-\frac{1}{2}} \left(\prod_{i=1}^{n} \sigma_i \right)^{-1} \\ &\quad \times \int_0^\infty \cdots \int_0^\infty \frac{\Phi(\widetilde{\mathbf{C}})}{\prod_{i=1}^{n} \widetilde{C}_i} \exp\left\{ -\frac{1}{2} \boldsymbol{\zeta}^T \mathbf{\Sigma}^{-1} \boldsymbol{\zeta} \right\} d\widetilde{\mathbf{C}},\end{aligned}$$

where $\mathbb{F}_0$ is the information history up to time 0, $\boldsymbol{\zeta} = (\zeta_1, \ldots, \zeta_n)'$, and $\zeta_i = \frac{1}{\sigma_i \sqrt{T}} \left(\ln\{\widetilde{C}_i / C_i(0)\} - (r - \frac{\sigma_i^2}{2}) T \right)$, $i = 1, \cdots, n$.

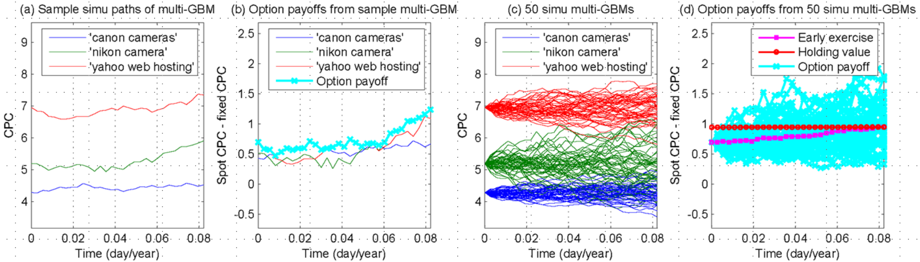
Solutions

# of keywords	Pricing method	Reference
1	Black-Scholes formula for European call option	See Appendix C
2	Peter Zhang dual strike European call option	See Appendix C
≥ 3	Monte Carlo simulation	See Algorithm 1

Algorithm 1:

```
function OptionPricingMC($K, C(0), \Sigma, M, m, r, T$)
    for $k \leftarrow 1$ to # of simulations do
        $[z_{1,k}, \ldots, z_{n,k}] \leftarrow$ GenerateMultivariateNoise$(MVN[0, M\Sigma M])$
        for $i \leftarrow 1$ to $n$ do
            $C_{i,k} \leftarrow C_i(0)\exp\left\{(r - \frac{1}{2}\sigma_i^2)T + \sigma_i z_{i,k}\sqrt{T}\right\}.$
        end for
        $G_k \leftarrow \Phi([C_{1,k}, \ldots, C_{n,k}]).$
    end for
    return $\pi_0 \leftarrow me^{-rT}\mathbb{E}_0[\Phi(C(T))] \approx me^{-rT}\left(\frac{1}{\tilde{n}}\sum_{k=1}^{\tilde{n}} G_k\right).$
end function
```

Empirical example of ad option pricing using Monte Carlo simulation



$$K = \left\{ \begin{array}{c} \text{‘canon cameras’} \\ \text{‘nikon camera’} \\ \text{‘yahoo web hosting’} \end{array} \right\}, \ \sigma = \begin{pmatrix} 0.2263 \\ 0.4521 \\ 0.2136 \end{pmatrix}, \ \boldsymbol{\Sigma} = \begin{pmatrix} 1.0000 & 0.2341 & 0.0242 \\ 0.2341 & 1.0000 & -0.0540 \\ 0.0242 & -0.0540 & 1.0000 \end{pmatrix}.$$

Revenue analysis for 1-keyword 1-click ad options

Let $D(F)$ be the difference between the expected revenue from an ad option and the expected revenue from only keyword auctions, we then have

$$
\begin{aligned}
D(F) = {} & \underbrace{\left(C(0)\mathcal{N}[\zeta_1] - e^{-rT}F\mathcal{N}[\zeta_2] + e^{-rT}F \right) \mathbb{P}(\mathbb{E}_0^{\mathbb{Q}}[C(T)] \geq F)}_{= \text{ Discounted value of expected revenue from option if } \mathbb{E}_0^{\mathbb{Q}}[C(T)] \geq F} \\
& + \underbrace{\left(C(0)\mathcal{N}[\zeta_1] - e^{-rT}F\mathcal{N}[\zeta_2] + e^{-rT}\mathbb{E}_0^{\mathbb{Q}}[C(T)] \right) \mathbb{P}(\mathbb{E}_0^{\mathbb{Q}}[C(T)] < F)}_{= \text{ Discounted value of expected revenue from option if } \mathbb{E}_0^{\mathbb{Q}}[C(T)] < F} \\
& - \underbrace{e^{-rT}\mathbb{E}_0^{\mathbb{Q}}[C(T)]}_{= \text{ Discounted value of expected revenue from auction}} . \\
= {} & C(0)\mathcal{N}[\zeta_1] - e^{-rT}F\mathcal{N}[\zeta_2] - e^{-rT}(\mathbb{E}_0^{\mathbb{Q}}[C(T)] - F) \times \mathbb{P}(\mathbb{E}_0^{\mathbb{Q}}[C(T)] \geq F),
\end{aligned}
$$

where $\mathcal{N}[\cdot]$ is the cumulative probability of a standard normal distribution.

Revenue analysis for 1-keyword 1-click ad options con't

- If $F = 0$, v_0 achieves its maximum value; therefore, $D(F) \to 0$.
- If $\pi_0 = 0$, F is as large as possible, $\mathbb{P}(\mathbb{E}_0^{\mathbb{Q}}[C(T)] \geq F) \to 0$ and $D(F) \to 0$.
- Since $\ln\{C(T)/C(0)\} \sim \mathbf{N}\big((r - \sigma^2/2)T, \sigma^2 T\big)$,

$$\mathbb{P}(\mathbb{E}_0^{\mathbb{Q}}[C(T)] \geq F) \approx \mathcal{N}\left[\frac{1}{\sigma\sqrt{T}}\Big(\ln\{C(0)/F\} + (r - \frac{1}{2}\sigma^2)T\Big)\right] = \mathcal{N}[\zeta_2].$$

Therefore, $D(F) = C(0)\mathcal{N}[\zeta_1](1 - e^{-\frac{1}{2}\sigma^2 T}) > 0$, then

$$\frac{\partial D(F)}{\partial F}\Big|_{F=\mathbb{E}_0^{\mathbb{Q}}[C(T)]} = 0, \quad \frac{\partial^2 D(F)}{\partial F^2}\Big|_{F=\mathbb{E}_0^{\mathbb{Q}}[C(T)]} = -\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}{\zeta_2}^2}\frac{1}{F\sigma\sqrt{T}} < 0,$$

suggests that by setting $F = \mathbb{E}_0^{\mathbb{Q}}[C(T)]$, the search engine can increase its profit.

Data[3]

Market	Group	Training set (31 days)	Deve&test set (31 days)
US	1	25/01/2012-24/02/2012	24/02/2012-25/03/2012
	2	30/03/2012-29/04/2012	29/04/2012-31/05/2012
	3	10/06/2012-12/07/2012	12/07/2012-17/08/2012
	4	10/11/2012-11/12/2012	11/12/2012-10/01/2013
UK	1	25/01/2012-24/02/2012	24/02/2012-25/03/2012
	2	30/03/2012-29/04/2012	29/04/2012-31/05/2012
	3	12/06/2012-13/07/2012	13/07/2012-19/08/2012
	4	18/10/2012-22/11/2012	22/11/2012-24/12/2012

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[3] The data was collected from Google AdWords by using its Traffic Estimation Service.

Validation conditions of the GBM model

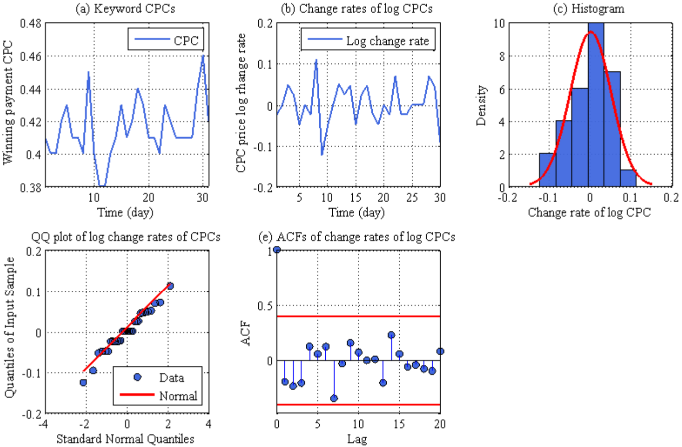
1. **Normality of change rates of log CPCs**

   Histogram/Q-Q plot and the Shapiro-Wilk test

2. **Independence from previous data**

   Autocorrelation function (ACF) and the Ljung-Box statistic

Empirical example of the GBM assumption test

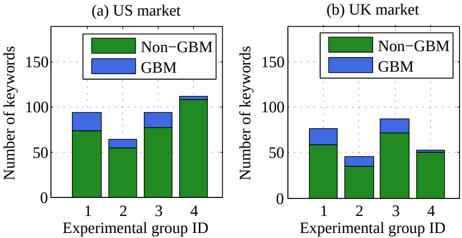


Keyword ‘canon 5d’

The Shapiro-Wilk test is with p-value 0.3712

The Ljung-Box test is with p-value 0.4555.

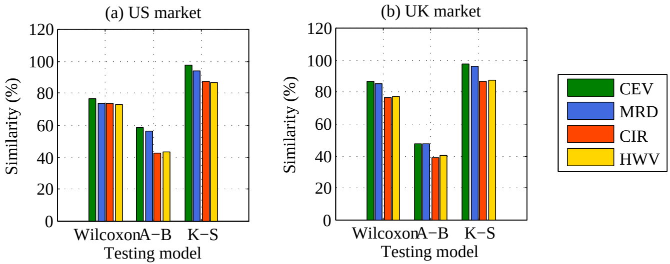
GBM assumption tests



There are 14.25% and 17.20% of keywords in US and UK markets that satisfy the GBM assumption, respectively.

Non-GBM dynamics and data fitting

Model	Stochastic differential equation (SDE)
Constant elasticity of variance (CEV)	$dC_i(t) = \mu_i C_i(t)dt + \sigma_i(C_i(t))^{1/2}dW_i(t)$
Mean-reverting drift (MRD)	$dC_i(t) = k_i(\mu_i - C_i(t))dt + \sigma_i(C_i(t))^{1/2}dW_i(t)$
Cox-Ingersoll-Ross (CIR)	$dC_i(t) = k_i(\mu_i - C_i(t))dt + (\sigma_i)^{1/2}C_i(t)dW_i(t)$
Hull-White/Vasicek (HWV)	$dC_i(t) = k_i(\mu_i - C_i(t))dt + \sigma_i dW_i(t)$



Wilcoxon test, Ansari-Bradley (A-B) test and Two-sample Kolmogorov-Smirnov (K-S) test

Check arbitrage via delta hedging

- Calculate delta
  - 1-keyword 1-click ad options

$$\frac{\partial V}{\partial C} = \mathcal{N}\left[\frac{1}{\sigma\sqrt{T}}\left(\ln\left\{\frac{C(0)}{F}\right\} + (r + \frac{\sigma^2}{2})T\right)\right].$$

  - n-keyword 1-click options (calculated by Monte Carlo method)

$$\partial V/\partial C_i = \mathbb{E}^{\mathbb{Q}}[\partial V(T, C(T))/\partial C_i(T)]$$

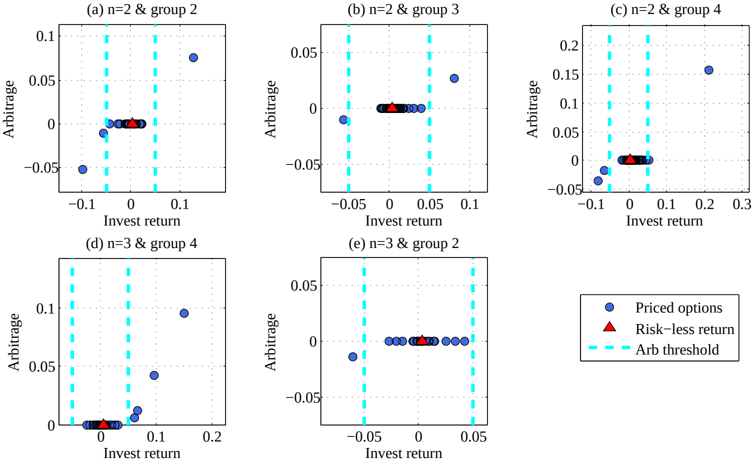
- The arbitrage detection criteria is

$$|\widetilde{\gamma} - \widetilde{r}| \leq \varepsilon \ ? \ \textbf{arbitrage does not exist} : \textbf{arbitrage exists},$$

where $\widetilde{\gamma}$ is the rate of returns from constructed hedging strategy, $\widetilde{r}$ is the equivalent (discrete) risk-less bank interest rate in the period, and ε is the model variation threshold (and we set $\varepsilon = 5\%$ in experiments). The identified arbitrage α is defined as the excess return, that is

$$\alpha = \begin{cases} \widetilde{\gamma} - (\widetilde{r} - \varepsilon), & \text{if } \widetilde{\gamma} < \widetilde{r} - \varepsilon, \\ \widetilde{\gamma} - (\widetilde{r} + \varepsilon), & \text{if } \widetilde{\gamma} > \widetilde{r} + \varepsilon. \end{cases}$$

Check arbitrage via delta hedging con't



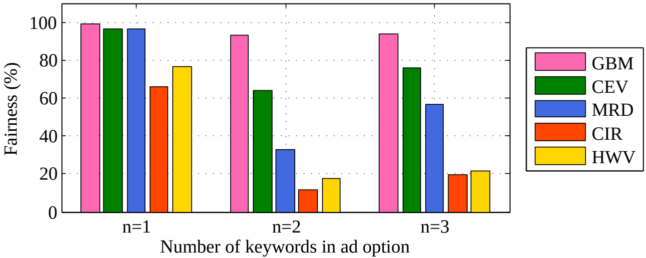
Empirical example of arbitrage analysis based on GBM for the US market.

Check arbitrage via delta hedging con't

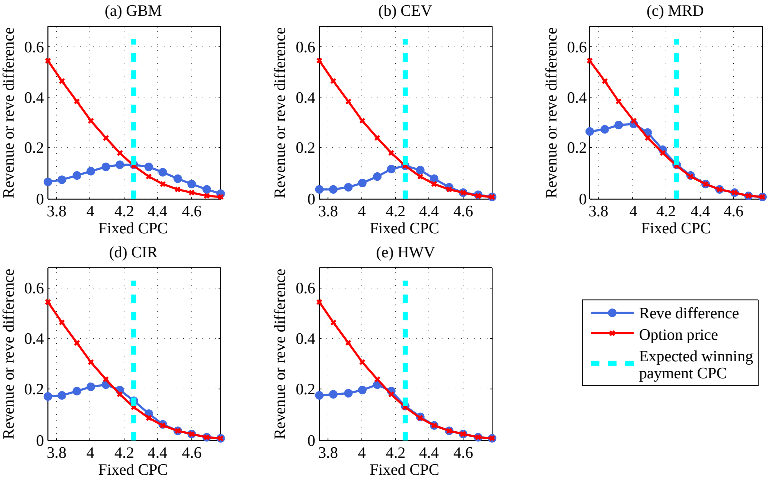
Testing arbitrage of options under the GBM: n is the number of keywords, N is the number of options priced in a group, $\mathbb{P}(\alpha)$ is % of options in a group identified arbitrage, and the $\mathbb{E}[\alpha]$ is the average arbitrage value of the options identified arbitrage.

n	Group	US market			UK market		
		N	$\mathbb{P}(\alpha)$	$\mathbb{E}[\alpha]$	N	$\mathbb{P}(\alpha)$	$\mathbb{E}[\alpha]$
1	1	94	0.00%	0.00%	76	0.00%	0.00%
	2	64	0.00%	0.00%	45	0.00%	0.00%
	3	94	1.06%	0.75%	87	0.00%	0.00%
	4	112	0.89%	-0.37%	53	0.00%	0.00%
2	1	47	4.26%	1.63%	38	0.00%	0.00%
	2	32	9.38%	0.42%	22	4.55%	13.41%
	3	47	4.26%	0.84%	43	4.65%	0.82%
	4	56	5.36%	3.44%	26	23.08%	-6.22%
3	1	31	0.00%	0.00%	25	4.00%	0.00%
	2	21	4.76%	-1.38%	15	0.00%	0.00%
	3	31	0.00%	0.00%	29	3.45%	-1.12%
	4	37	10.81%	3.87%	17	35.29%	-2.54%

Robust tests of pricing models

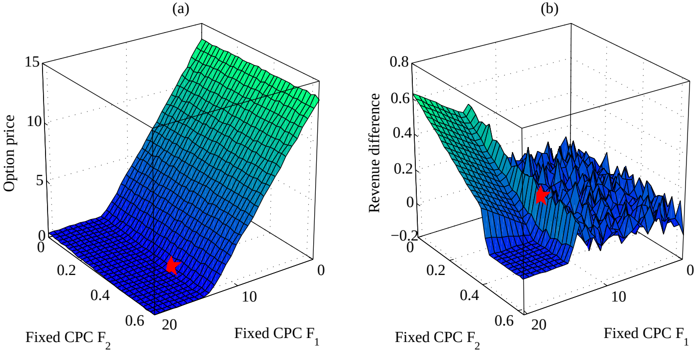


Empirical example of revenue analysis for a 1-keyword 1-click ad option



Keyword ‘canon cameras’

Empirical example of revenue analysis for a 2-keyword 1-click ad option



Keywords ‘non profit debt consolidation’ and ‘canon 5d’, where $\rho = 0.2247$

Limitations and future work

- Other sophisticated stochastic processes are worth studying, such as jump-diffusion models and stochastic volatility models.
- Game-theoretical pricing models for ad options.
- Optimal pricing and allocation of ad options.